Paper Reference(s)

6679

Edexcel GCE

Mechanics M3

Advanced Subsidiary

Thursday 14 June 2007 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

N26116A

1. The rudder on a ship is modelled as a uniform plane lamina having the same shape as the region R which is enclosed between the curve with equation $y = 2x - x^2$ and the x-axis.

(*a*) Show that the area of R is $\frac{4}{3}$.

(4)

(*b*) Find the coordinates of the centre of mass of the lamina.

(5)

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2. An open container C is modelled as a thin uniform hollow cylinder of radius h and height h with a base but no lid. The centre of the base is O.

(*a*) Show that the distance of the centre of mass of C from O is $\frac{1}{3}h$.

(5)

The container is filled with uniform liquid. Given that the mass of the container is M and the mass of the liquid is M,

(*b*) find the distance of the centre of mass of the filled container from O.

(5)

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3. A spacecraft S of mass m moves in a straight line towards the centre of the earth. The earth is modelled as a fixed sphere of radius R. When S is at a distance x from the centre of the earth, the force exerted by the earth on S is directed towards the centre of the earth and has magnitude $\frac{k}{x^2}$, where k is a constant.

(*a*) Show that $k = mgR^2$.

(2)

Given that S starts from rest when its distance from the centre of the earth is $2R$, and that air resistance can be ignored,

(*b*) find the speed of S as it crashes into the surface of the earth.

(7)

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4. A light inextensible string of length l has one end attached to a fixed point A. The other end is attached to a particle P of mass m. The particle moves with constant speed v in a horizontal circle with the string taut. The centre of the circle is vertically below A and the radius of the circle is r.

Show that

$$gr^2 = v^2\sqrt{(l^2 - r^2)}.$$

(9)

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5. A particle P moves on the x-axis with simple harmonic motion about the origin O as centre. When P is a distance 0.04 m from O, its speed is 0.2 m s^{-1} and the magnitude of its acceleration is 1 m s^{-2}.

(*a*) Find the period of the motion.

(3)

The amplitude of the motion is a metres.

Find

(*b*) the value of a,

(3)

(*c*) the total time, within one complete oscillation, for which the distance OP is greater than $\frac{1}{2}a$ metres.

(5)

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6. A particle P is free to move on the smooth inner surface of a fixed thin hollow sphere of internal radius a and centre O. The particle passes through the lowest point of the spherical surface with speed U. The particle loses contact with the surface when OP is inclined at an angle α to the upward vertical.

(*a*) Show that $U^2 = ag(2 + 3\cos\alpha)$.

(7)

The particle has speed W as it passes through the level of O. Given that $\cos\alpha = \dfrac{1}{\sqrt{3}}$,

(*b*) show that

$$W^2 = ag\sqrt{3}.$$

(5)

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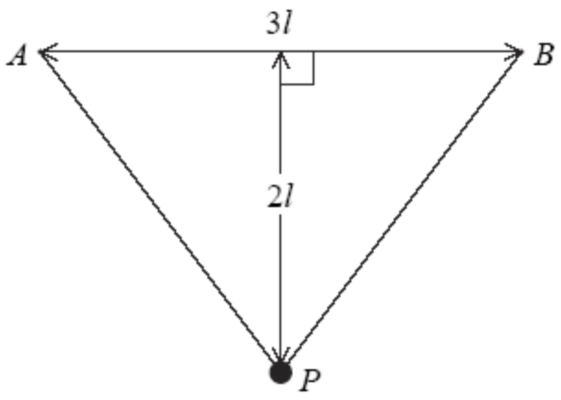
7. **Figure 1**



A light elastic string, of natural length $3l$ and modulus of elasticity λ, has its ends attached to two points A and B, where $AB = 3l$ and AB is horizontal. A particle P of mass m is attached to the mid-point of the string. Given that P rests in equilibrium at a distance $2l$ below AB, as shown in Figure 1,

(*a*) show that $\lambda = \dfrac{15mg}{16}$.

(9)

The particle is pulled vertically downwards from its equilibrium position until the total length of the elastic string is $7.8l$. The particle is released from rest.

(*b*) Show that P comes to instantaneous rest on the line AB.

(6)

TOTAL FOR PAPER: 75 MARKS

END

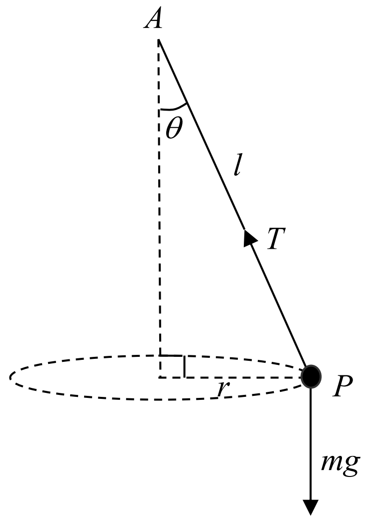
edexcel

June 2007
6679 Mechanics M3
Mark Scheme

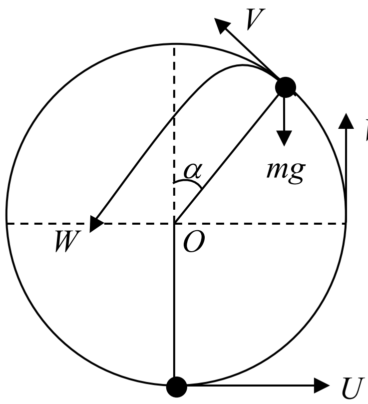
Question Number	Scheme	Marks
1.	(a) $A=\int_0^2 \left(2x-x^2\right)\mathrm{d}x$	M1 A1
	$=\left[x^2-\frac{x^3}{3}\right]_{...}^{...}$	A1
	$A=\left[x^2-\frac{x^3}{3}\right]_0^2=4-\frac{8}{3}=\frac{4}{3}$ * cso	A1 **(4)**
	(b) $\bar{x}=1$ (by symmetry)	B1
	$\frac{4}{3}\bar{y}=\frac{1}{2}\int y^2\,\mathrm{d}x=\frac{1}{2}\int\left(2x-x^2\right)^2\mathrm{d}x$	M1
	$=\frac{1}{2}\int\left(4x^2-4x^3+x^4\right)\mathrm{d}x$	A1
	$=\frac{1}{2}\left[\frac{4x^3}{3}-x^4+\frac{x^5}{5}\right]$	A1
	$\frac{4}{3}\bar{y}=\frac{1}{2}\left[\frac{4x^3}{3}-x^4+\frac{x^5}{5}\right]_0^2=\frac{8}{15}$	
	$\bar{y}=\frac{8}{15}\times\frac{3}{4}=\frac{2}{5}$ accept exact equivalents	A1 **(5)**
		[9]

Question Number	Scheme	Marks
2.	(a) Base / Cylinder / Container	
	Mass ratios: πh^2, $2\pi h^2$, $3\pi h^2$ — Ratio of 1 : 2 : 3	B1
	$\bar{y}$: 0, $\frac{h}{2}$, $\bar{y}$	B1
	$3\pi h^2 \times \bar{y} = 2\pi h^2 \times \frac{h}{2}$	M1 A1
	Leading to $\bar{y} = \frac{1}{3}h$ ✱ — cso	A1 **(5)**
	(b) Liquid / Container / Total	
	Mass ratios: M, M, $2M$ — Ratio of 1 : 1 : 2	B1
	$\bar{y}$: $\frac{h}{2}$, $\frac{h}{3}$, $\bar{y}$	B1
	$2M \times \bar{y} = M \times \frac{h}{2} + M \times \frac{h}{3}$	M1 A1
	$\bar{y} = \frac{5}{12}h$	A1 **(5)**
		[10]

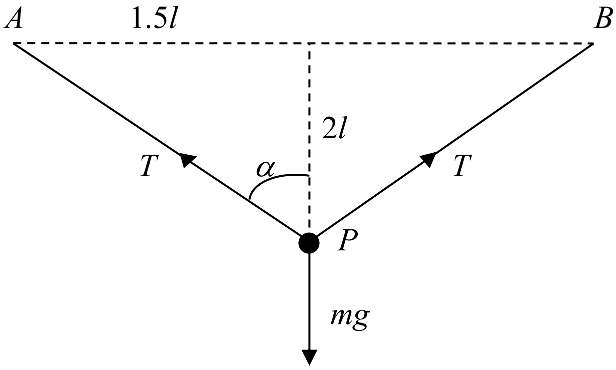
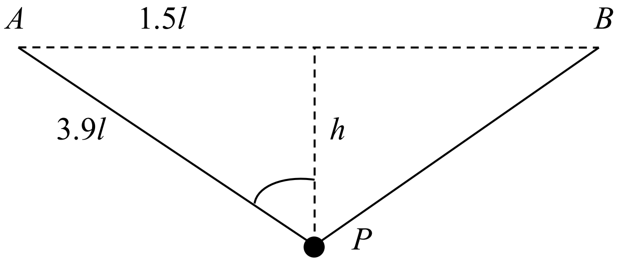
Question Number	Scheme	Marks
3.	(a) At surface	
	$\frac{k}{R^2} = mg \quad \Rightarrow \quad k = mgR^2$ * cso	M1 A1 **(2)**
	(b) N2L $\quad m\ddot{x} = -\frac{mgR^2}{x^2}$	
	$v\frac{\mathrm{d}v}{\mathrm{d}x} = -\frac{gR^2}{x^2}$ or $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{2}v^2\right) = -\frac{gR^2}{x^2}$	M1
	$\int v\,\mathrm{d}v = -gR^2\int\frac{1}{x^2}\,\mathrm{d}x$ or $\frac{1}{2}v^2 = -gR^2\int\frac{1}{x^2}\,\mathrm{d}x$	M1
	$\frac{1}{2}v^2 = \frac{gR^2}{x} \quad (+C)$	A1
	$x = 2R, v = 0 \quad \Rightarrow \quad C = -\frac{gR}{2}$	M1 A1
	$v^2 = \frac{2gR^2}{x} - gR$	
	At $x = R$, $\quad v^2 = \frac{2gR^2}{R} - gR$	M1
	$v = \sqrt{(gR)}$	A1 **(7)**
		[9]

Question Number	Scheme		Marks	
4.				
	↑ $T\cos\theta = mg$		M1 A1	
	← $T\sin\theta = \dfrac{mv^2}{r}$		M1 A1	
	$\tan\theta = \dfrac{r}{\sqrt{(l^2 - r^2)}}$	or equivalent	M1 A1	
	$\tan\theta = \dfrac{v^2}{rg}$	Eliminating T	M1	
	$\dfrac{r}{\sqrt{(l^2 - r^2)}} = \dfrac{v^2}{rg}$	Eliminating θ	M1	
	$gr^2 = v^2\sqrt{(l^2 - r^2)}$ *	cso	A1	**(9)**
				[9]

Question Number	Scheme	Marks
5.	(a) $\ddot{x} = -\omega^2 x \quad \Rightarrow \quad 1 = \omega^2 \times 0.04 \quad (\Rightarrow \quad \omega = 5)$	M1 A1
	$T = \frac{2\pi}{5}$ awrt 1.3	A1 **(3)**
	(b) $v^2 = \omega^2 \left(a^2 - x^2\right) \quad \Rightarrow \quad 0.2^2 = 5^2 \left(a^2 - 0.04^2\right)$ ft their ω	M1 A1ft
	$a = \frac{\sqrt{2}}{25}$ accept exact equivalents or awrt 0.057	A1 **(3)**
	(c) Using $x = a\cos\omega t$	
	$\frac{1}{2}a = a\cos\omega t$ ft their ω	M1 A1ft
	$5t = \frac{\pi}{3}$	
	$t = \frac{\pi}{15}$	A1
	$T' = 4t = \frac{4\pi}{15}$ awrt 0.84	M1 A1 **(5)**
		[11]
	Alternative to (c)	
	Using $x = a\sin\omega t$	
	$\frac{1}{2}a = a\sin\omega t$ ft their ω	M1 A1ft
	$5t = \frac{\pi}{6}$	
	$t = \frac{\pi}{30}$	A1
	$T' = T - 4t = \frac{4\pi}{15}$ awrt 0.84	M1 A1 **(5)**

Question Number	Scheme	Marks
6.		
	(a) Energy $\frac{1}{2}m\left(U^2 - v^2\right) = mga\left(1+\cos\alpha\right)$	M1 A1=A1
	$\swarrow$ $\left(T+\right)$ $mg\cos\alpha = \frac{mv^2}{a}$	M1 A1
	Leaves circle when $T = 0$	
	$g\cos\alpha = \frac{U^2 - 2ga - 2ga\cos\alpha}{a}$ Eliminating v	M1
	Leading to $U^2 = ag\left(2+3\cos\alpha\right)$ ∗ cso	A1 **(7)**
	(b) Using conservation of energy from the lowest point of the surface	
	$\frac{1}{2}m\left(U^2 - W^2\right) = mga$	M1 A1=A1
	$W^2 = U^2 - 2ag$	
	Using $\cos\alpha = \frac{1}{\sqrt{3}}$, $W^2 = ag\left(2+\frac{3}{\sqrt{3}}\right) - 2ag$	M1
	$= ag\sqrt{3}$ ∗ cso	A1 **(5)**
		[12]
	Alternatives for (b) *are given on the next page.*	

Question Number	Scheme	Marks
6.	*Alternative to part* (b) *using conservation of energy from the point where P loses contact with surface.*	
	$\left(V^2 = ag\cos\alpha = \dfrac{ga}{\sqrt{3}}\right)$	
	Energy $\quad \frac{1}{2}m\left(W^2 - V^2\right) = mga\cos\alpha$	M1 A1
	$\frac{1}{2}m\left(W^2 - \frac{1}{\sqrt{3}}ag\right) = mga \times \frac{1}{\sqrt{3}}$	A1
	Leading to $\quad W^2 = ag\sqrt{3}$ * $\quad$ cso	M1 A1 **(5)**
	Alternative to part (b) *using projectile motion from the point where P loses contact with surface.*	
	$V^2 = ag\cos\alpha = \dfrac{ga}{\sqrt{3}}$	
	$\downarrow \quad W_y^2 = V^2\sin^2\alpha + 2ga\cos\alpha$	
	$= \frac{1}{\sqrt{3}}ag\left(1 - \frac{1}{3}\right) + 2ga \times \frac{1}{\sqrt{3}} = \frac{8\sqrt{3}}{9}ag$	M1 A1
	$\leftarrow \quad V_x = V\cos\alpha$	A1
	$W^2 = W_y^2 + V_x^2 = \frac{8\sqrt{3}}{9}ag + \frac{1}{3}ag\sqrt{3} \times \frac{1}{3} = ag\sqrt{3}$ * $\quad$ cso	M1 A1 **(5)**

Question Number	Scheme	Marks
7.	(a) 	
	$AP = \sqrt{\left((1.5l)^2 + (2l)^2\right)} = 2.5l$	M1 A1
	$\cos\alpha = \frac{4}{5}$	B1
	Hooke's Law $T = \frac{\lambda(2.5l - 1.5l)}{1.5l} \left(= \frac{2\lambda}{3}\right)$	M1 A1
	↑ $2T\cos\alpha = mg \qquad \left(T = \frac{5mg}{8}\right)$	M1 A1
	$2 \times \frac{2\lambda}{3} \times \frac{4}{5} = mg \qquad \left(\frac{2\lambda}{3} = \frac{5mg}{8}\right)$	M1
	$\lambda = \frac{15mg}{16}$ ∗ cso	A1 **(9)**
	(b) 	
	$h = \sqrt{\left((3.9l)^2 - (1.5l)^2\right)} = 3.6l$	M1 A1
	Energy $\frac{1}{2}mv^2 + mg \times h = 2 \times \frac{15mg}{16} \times \frac{(2.4l)^2}{2 \times 1.5l}$ ft their h	M1 A1ft = A1
	Leading to $v = 0$ ∗ cso	A1 **(6)** **[15]**